Contextuality: from impossible figures to quantum correlations

Pierre-Emmanuel Emeriau and Shane Mansfield

Contextuality

It is typically the case that not all properties of a quantum system may be observed at once: so that at best we can only ever obtain partial “snapshots” of the system by observing some of its properties at once.

Contextuality arises when these snapshots are locally consistent — they agree whenever they overlap — but they are nevertheless globally inconsistent.

A very useful analogy for this is the Penrose staircase, as depicted in M.C. Escher’s famous Ascending and descending lithograph.

The Penrose Staircase

Where snapshots overlap the information is locally consistent, but if we try to piece the snapshots together to obtain a global picture we find a global inconsistency (the impossible staircase, left)

Another example: the tribar

A quantum experiment

The real world is contextual?!

The empirical data above is inconsistent with the assumption that observable properties have definite values which do not depend on which observations will be made subsequently.

• E.g. it might be the case that \( |a, a, b, b, b\rangle\) all take value 0, which would correspond to bundle diagram (c); or it might be that \( |a, a, b, b\rangle\) all take value 1, which would correspond to bundle diagram (d).

• It might even be that the preparation sometimes results in (c) and sometimes in (d) in which case we could end up with the bundle diagram (e)

The empirical data

Probability table for empirical observations: e.g. the top left entry tells us that there is probability \( \frac{3}{4} \) of seeing outcomes \( a \) and \( b \) when red choses to observe \( a \) and blue to observe \( b \).

Visualising empirical data

• Bundle diagrams allow us to visualise empirical data in terms of which combinations of observational events are possible and which are impossible.

• Diagram (a) is a visualisation of the empirical data table above.

• It tells us that when the observables are \( a, b \) the only possible events are \( (a, a) \) and \( (1, 1) \), and so on.

Classical data

• In the classical world, once a system is prepared its observable properties have definite values which do not depend on which observations will be made subsequently

• E.g. it might be the case that \( |a, a, b, b\rangle\) all take value 0, which would correspond to bundle diagram (c); or it might be that \( |a, a, b, b\rangle\) all take value 1, which would correspond to bundle diagram (d).

• It might even be that the preparation sometimes results in (c) and sometimes in (d) in which case we could end up with the bundle diagram (e)

The empirical data above is inconsistent with the assumption that observable properties have definite values independent of observational context — see diagram (b).

• Observations can only give us partial snapshots of quantum reality. As with the Penrose staircase, the snapshots are locally consistent, but when pieced together to form a global picture of this underlying reality they are globally inconsistent.

• It is typically the case in such a quantum experiment that one can filter out some fraction of the data which can be explained classically, but can still be left with a large portion of data as in the table and diagrams (a) and (b) above which cannot